Centre Number

Student Number

**SCEGGS Darlinghurst**

**2008**

HIGHER SCHOOL CERTIFICATE
TRIAL EXAMINATION

# Mathematics

This is a TRIAL PAPER only and does not necessarily reflect the content or format of the Higher School Certificate Examination for this subject.

**General Instructions**

- Reading time – 5 minutes
- Working time – 3 hours
- Write using black or blue pen
- Diagrams should be drawn in pencil
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- All necessary working should be shown in every question
- Start each question in a new booklet

**Total marks – 120**

- Attempt Questions 1–10
- All questions are of equal value

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**Total marks – 120**
**Attempt Questions 1–10**
**All questions are of equal value**

Answer each question in a SEPARATE writing booklet. Extra writing booklets are available.

**Marks**

**Question 1** (12 marks)

(a) Find the value of $\sqrt{\dfrac{19}{4\pi}}$ correct to two decimal places. **2**

(b) Solve $x - 3(2 - x) = 0$. **2**

(c) Factorise $a^3 - 27$. **2**

(d) Write $0.5\dot{2}\dot{6}$ in the form of $\dfrac{a}{b}$. **2**

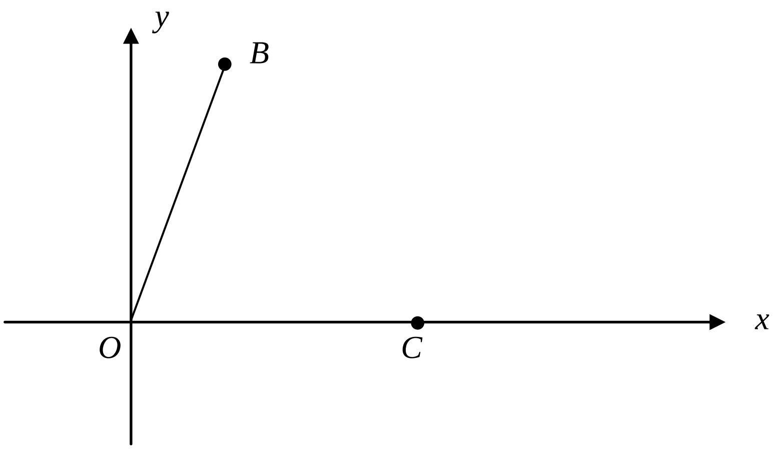
(e) Kate invests \$5500 for 3 years. The investment earns 7.5% p.a. compounded annually. How much interest does Kate's investment earn after 3 years? **2**

(f) Find a primitive function of $4 - \dfrac{1}{x^2}$ . **2**

**Marks**

**Question 2** (12 marks) Use a SEPARATE writing booklet.

(a)




O is the origin, $B$ is the point $(1, 6)$ and $C$ is the point $(5, 0)$.

Copy this diagram into your writing booklet.

(i) Find the co-ordinates of the point $P$ such that $OBPC$ is a parallelogram. Label this point on your diagram. **1**

(ii) Find, in its simplified form, the exact length of $BC$. **2**

(iii) Find the area of the parallelogram $OBPC$. **1**

(iv) Hence or otherwise, find the perpendicular distance from $P$ to the diagonal $BC$. **2**

(v) Find $\angle BCO$ to the nearest degree. **2**

(b) The quadratic equation $4x^2 - 3x + 1 = 0$ has roots $\alpha$ and $\beta$. Find:

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\dfrac{1}{\alpha^2} + \dfrac{1}{\beta^2}$ **2**

**Marks**

**Question 3** (12 marks) Use a SEPARATE writing booklet.

(a) Find:

(i) $\frac{d}{dx}\left(\sin 3x\right)$ **2**

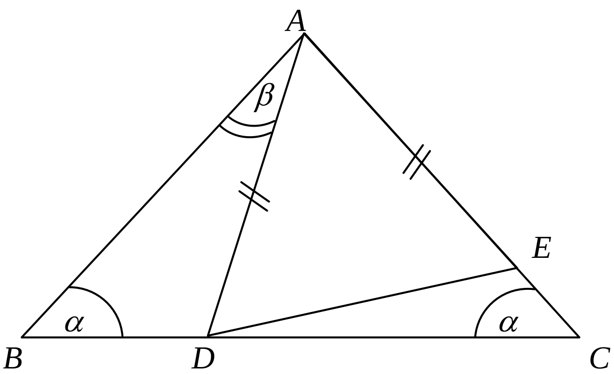
(ii) $\frac{d}{dx}\left(x^2 e^x\right)$ **2**

(b) Evaluate each of the following correct to 3 decimal places:

(i) $\int_0^1 e^{-4x}\, dx$ **2**

(ii) $\int_1^5 \frac{dx}{\sqrt{x}}$ **2**

(c)




In the isosceles triangle $ABC$, $\angle ABC = \angle ACB = \alpha$. The points $D$ and $E$ lie on $BC$ and $AC$, so that $AE = AD$. Let $\angle BAD = \beta$.

(i) Explain why $\angle ADC = \alpha + \beta$. **1**

(ii) Find $\angle DAC$ in terms of $\alpha$ and $\beta$. Give reasons. **1**

(iii) Hence, or otherwise, find $\angle EDC$ in terms of $\beta$. **2**

**Marks**

**Question 4** (12 marks) Use a SEPARATE writing booklet.

(a) The movie theatre on the QEII is specially designed. The floor narrows from front to back so that each row of seats behind the first has two seats less than the row in front of it. The front row has 37 seats.

(i) How many seats are in the $n^{\text{th}}$ row? **2**

(ii) What is the greatest value $n$ can take? **1**

(iii) The movie theatre seats 360 people. How many rows of seats are there? **3**

(b) A function is defined as $y = \ln x$.

(i) Copy the table into your writing book and complete it giving answers to 3 decimal places. **1**

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y = \ln x$ | | | | |

(ii) Using the trapezoidal rule, find an approximation, to 1 decimal place, of $\int_2^5 \ln x \, dx$ using 4 function values. **3**

(iii) Sketch the graph of $y = \ln x$ and use it to explain why your approximation in part (ii) will be less than the exact value of the integral. **2**

**Marks**

**Question 5** (12 marks) Use a SEPARATE writing booklet.

(a) (i) Sketch $y = 2\sin 2x$ for $0 \le x \le 2\pi$. **2**

(ii) Calculate the area bounded by the curve $y = 2\sin 2x$ and the $x$-axis between $x = 0$ and $x = \dfrac{3\pi}{4}$. **3**

(b) A box contains 4 red and 5 green marbles. Anna randomly selects 3 marbles one at a time and without replacement.

(i) What is the probability that she selects green, red, green in that order? **1**

(ii) What is the probability that she selects a majority of green marbles? **2**

(c) (i) If $f(x) = \log_e \sqrt{x}$ evaluate $f(e^2)$. **1**

(ii) Find the equation of the tangent to the curve $y = \log_e \sqrt{x}$ at the point where $x = e^2$. **3**

**Marks**

**Question 6** (12 marks) Use a SEPARATE writing booklet.

(a) For the curve $y = x^3 - 3x$:

(i) Find the stationary points and determine the nature of these points. **3**

(ii) Show that $(0, 0)$ is a point of inflexion. **1**

(iii) Sketch the curve for $-2 \le x \le 3$. **2**

(iv) Hence, or otherwise, evaluate $\int_{-1}^{1} x^3 - 3x \, dx$ **1**

(b) Carl the caterpillar woke up one morning and he was feeling very hungry. So he found himself a big leaf and started munching. After 10 minutes he had eaten 5cm of the leaf. Since his tummy was starting to fill up, in the next 10 minutes he ate 3.75 cm of the leaf. In the next 10 minutes he ate 2.8125 cm of the leaf and so on. **2**

If the leaf was 25 cm long, would Carl finish eating the leaf for breakfast? Give reasons.

(c) (i) Find $\frac{d}{dx}\left(e^{\tan 2x}\right)$. **1**

(ii) Hence, evaluate $\int_{0}^{\frac{\pi}{6}} \left(\sec^2 2x\right) e^{\tan 2x}$ . Leave your answer in exact form. **2**

**Marks**

**Question 7** (12 marks) Use a SEPARATE writing booklet.

(a) For the parabola $y^2 - 2y - 4x - 7 = 0$, find:

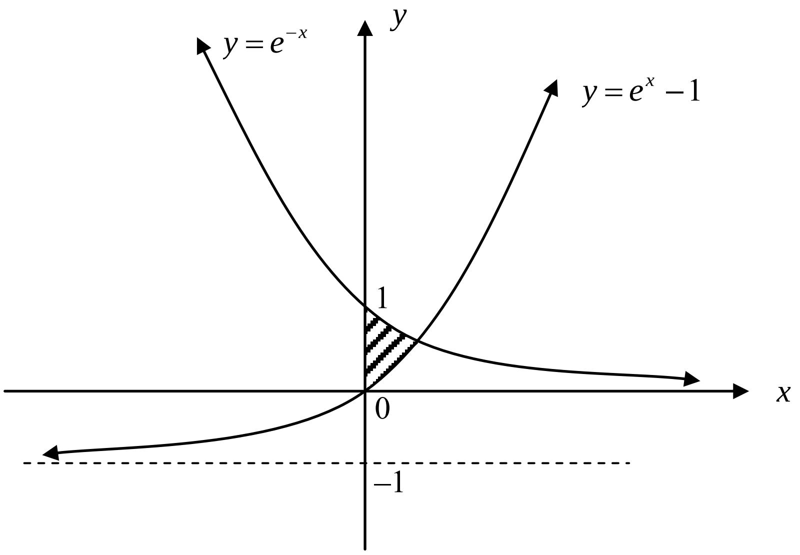
(i) the co-ordinates of the vertex. **2**

(ii) the co-ordinates of the focus. **1**

(iii) the equation of the directrix. **1**

(iv) Sketch $y^2 - 2y - 4x - 7 = 0$, labelling the vertex, focus and directrix only. **2**

(b) The diagram shows the graph of $y = e^x - 1$ and $y = e^{-x}$.




(i) Show that the curves intersect when: **2**

$$e^{2x} - e^x - 1 = 0$$

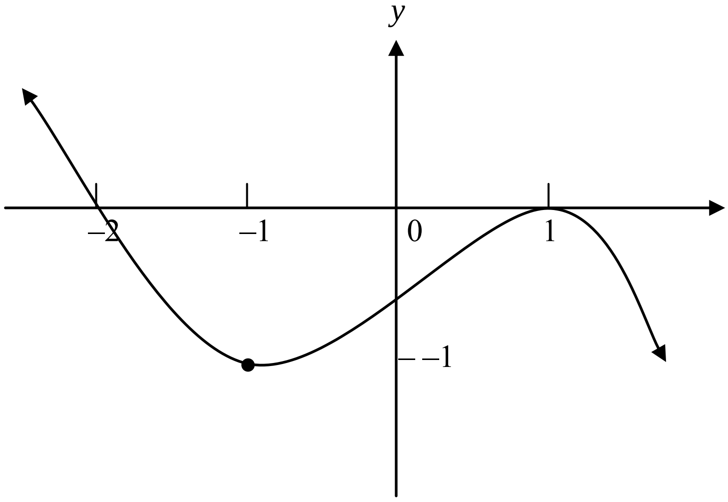
(ii) Use the results in part (i) and the substitution $m = e^x$, to show that the $x$ - co-ordinate of the point of intersection of the curves is approximately 0.481. **2**

(iii) Find the area of the shaded region to 1 decimal place. **2**

**Marks**

**Question 8** (12 marks) Use a SEPARATE writing booklet.

(a) The graph below shows $y = f'(x)$.




Copy the graph into your writing booklet.

(i) For which values of $x$ is the curve of $y = f(x)$ increasing. **1**

(ii) On the same axes sketch $y = f(x)$ clearly labelling the important features at $x = -2, x = -1$ and $x = 1$. **3**

(b) (i) For what values of $k$ does the quadratic equation $kx^2 + (k+3)x - 1 = 0$ have no real roots. **3**

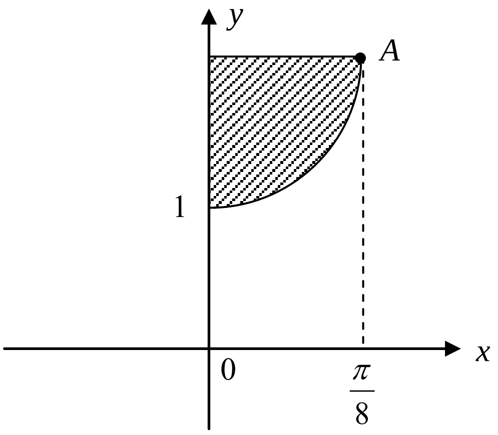
(ii) Hence, explain why $y = kx^2 + (k+3)x - 1$ can never be positive definite. **1**

**Question 8 continues on page 10**

**Marks**

Question 8 (continued)

(c) The graph is $y = \sec 2x$ from $x = 0$ to $x = \frac{\pi}{8}$.




(i) The $x$-coordinate of $A$ is $\frac{\pi}{8}$. Find the $y$-coordinate. **1**

(ii) The shaded region is rotated around the $x$-axis. **3**
Show that the volume generated by this rotation is given by:

$$\frac{\pi^2}{4} - \pi\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$$

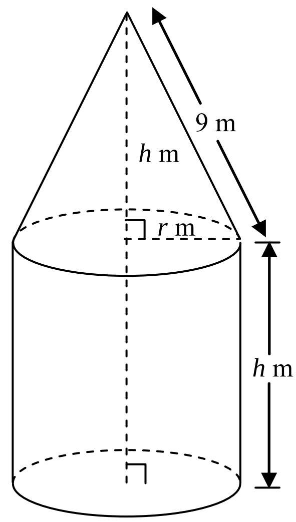
and hence find the exact volume.

**Marks**

**Question 9** (12 marks) Use a SEPARATE writing booklet.

(a) Solve $2\sin^2 x - 3\cos x = 0$ for $0 \le x \le 2\pi$. **3**

(b) A grain silo is constructed from a right circular cone and cylinder of equal heights "$h$" m. The slant edge of the cone is 9 m and the radius of the cylinder is "$r$" m.




(i) Write down the equation linking $r$ and $h$. **1**

(ii) Show that the volume of the silo is given by: **2**

$$V = 108\pi h - \frac{4}{3}\pi h^3$$

(iii) Find the height of the silo that gives a maximum volume. **3**

(c) In the Jackpot Lottery, the probability of the Jackpot prize being won in any draw is approximately 1 in 50.

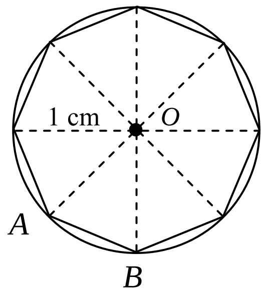
(i) What is the probability that the Jackpot prize will be won in each of three consecutive draws? **1**

(ii) How many consecutive draws must be made for there to be a 50% chance that at least one Jackpot prize will have been won? **2**

**Marks**

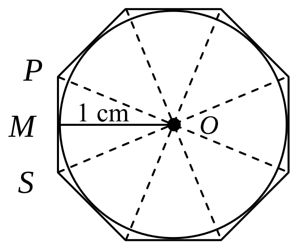
**Question 10** (12 marks) Use a SEPARATE writing booklet.

(a) A regular octagon is drawn inside a circle with centre O so that its vertices lie on the circumference. The circle has radius 1 cm.




(i) Find the area of $\Delta AOB$ and hence find the area of this octagon. (Leave your answer in surd form.) **2**

Another regular octagon is drawn outside the circle.
The altitude $OM$ of $\Delta OPS$ is 1 cm.




(ii) Find the area of $\Delta OPS$ and hence find the area of this outer octagon. **2**

(iii) By considering the results in parts (i) and (ii), show that: **1**

$$\sqrt{2} < \frac{\pi}{2} < 4\tan\frac{\pi}{8}$$

**Question 10 continues on page 13**

**Marks**

Question 10 (continued)

(b) Miss Toolan decides to borrow \$400 000 to buy an apartment. Interest is calculated monthly on the balance still owing, at a rate of 9.12% p.a. The loan is to be repaid at the end of 25 years with equally monthly repayments of $\$M$. Let $\$\,A_n$ be the amount owing after the $n^{\text{th}}$ repayment.

(i) Derive an expression for $A_{300}$. **2**

(ii) Find the value of $M$. **2**

(iii) Calculate the amount still owing, to the nearest dollar, after 10 years of payments at this rate. **1**

(iv) At the end of 10 years, the interest rate is increased to 12% p.a. and Miss Toolan changes her repayments to \$3800 per month. **2**

How many months are needed to pay off the remainder of the loan?

**End of Paper**

# STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln\left(x + \sqrt{x^2 - a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln\left(x + \sqrt{x^2 + a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

2008 Trial HSC MATHEMATICS – Solutions

**Q1** a) 1·23 ✓ & 1 for correct rounding.

Done very well

b) $x - 3(2-x) = 0$

$x - 6 + 3x = 0$ ✓

$4x - 6 = 0$

$4x = 6$

$x = \frac{6}{4}$

$= \frac{3}{2}$ ✓

Quite a few students made silly errors. The most annoying was thinking it was a quadratic $(x-3)(2-x)=0$

c) $a^3 - 27 = (a-3)(a^2+3a+9)$ ✓ ✓

Done fairly well, most common mistake $a^2+6a+9$

d) let $x = 0\cdot5262626\ldots$

$10x = 5\cdot262626\ldots$ ① ✓

$1000x = 526\cdot262626\ldots$ ②

② − ① $990x = 521$

$x = \frac{521}{990}$ ✓

Done well. Most errors came from not aligning the recurring decimals i.e. 52·6.... 526·2....

e) $A = P\left(1+\frac{r}{100}\right)^n$

$= 5500\left(1+\frac{7\cdot5}{100}\right)^3$ ✓

$= 6832\cdot63$

Int $= 6832\cdot63 - 5500$

$= 1332\cdot63$

Kate receives \$1332·63 interest. ✓

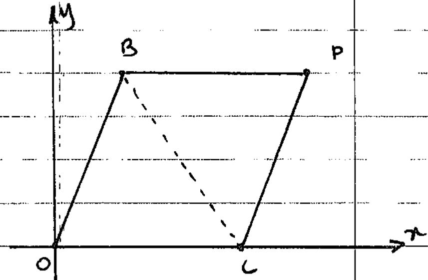
Most common mistake was not reading the question. It asked for the interest not how much it was worth

f) $4x + x^{-1}$

$= 4x + \frac{1}{x} + c$ ✓ ✓ Calc – 2

Done well

**Q2**




i) $P(6,6)$ ✓

ii) $BC = \sqrt{(5-1)^2+(0-6)^2}$ ✓

$= \sqrt{4^2+(-6)^2}$

$= \sqrt{16+36}$

$= \sqrt{52}$

$= 2\sqrt{13}$ units ✓

Many students did not 'update' their diagram at a... This is why students are asked to copy in the fi... place! Many students ass... they were dealing with a rhombus (because it looks liked one) The diagonals of this shape do NOT bis... at right angles. A few students did not underst... that the vertices should ... listed cyclically and pla... P in the wrong position.

iii) $A = l \times h$

$= 5 \times 6$

$= 30$ units$^2$ ✓

Many responses were very long and complicated. Be guided by the marking scheme.

iv) Area of //ogram OBPC = Area $\Delta OBC$ + Area $\Delta CBP$

$30 = 2 \times$ Area $\Delta CBP$ $\quad \Delta OBC \equiv \Delta CBP$

$30 = 2 \times \frac{1}{2} \times BC \times \perp$ dist from P ✓

$\perp$ dist from P $= \frac{30}{2\sqrt{13}}$

$= \frac{15}{\sqrt{13}}$ units. ✓ Reas – 2

Some students who used the distance or other formulas, made careless substitution errors

v) $\sin \angle CBP = \dfrac{\frac{15}{\sqrt{13}}}{5}$ ✓

$= 56°$ (nearest degree) ✓

$\angle BCO = 56°$ (alternate angles in parallel lines)

Reas – 2

Many students introduce... an angle θ without upda... it on their diagram. The marker has no knowledg... of where θ is. Similarly, very few students correctly used m = tan θ and identi... the acute angle

Q2 cont. b) i) $\alpha+\beta = \frac{-b}{a}$

$= \frac{-(-3)}{4}$

$= \frac{3}{4}$ ✓

ii) $\alpha\beta = \frac{c}{a}$

$= \frac{1}{4}$ ✓

iii) $\frac{1}{\alpha^2} + \frac{1}{\beta^2} = \frac{\alpha^2+\beta^2}{\alpha^2\beta^2}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{(\alpha\beta)^2}$ ✓

$= \frac{\left(\frac{3}{4}\right)^2 - 2\times\frac{1}{4}}{\left(\frac{1}{4}\right)^2}$

$= \frac{\frac{9}{16} - \frac{1}{2}}{\frac{1}{16}}$

$= 1$ ✓

Few students could handl[e] the algebra. Many had learned off that $(\alpha+\beta)^2 = \alpha^2+\beta^2-2\alpha\beta$ but ha[d] no idea how to apply it!

Q3 a) i) $\frac{d}{dx}(\sin 3x)$

$= 3\cos 3x$ ✓ ✓

ii) $\frac{d}{dx}(x^2e^x) = x^2e^x + 2xe^x$ ✓

$= e^x(x^2+2x)$ ✓ Calc - 4

On the whole well done. Some mix-up with integration. Funny to see "+C" at this question. Well done.

b) i) $\int_0^1 e^{-4x}\,dx = -\frac{1}{4}e^{-4x}\Big]_0^1$ ✓

$= -\frac{1}{4}e^{-4} + \frac{1}{4}e^0$

$= -\frac{1}{4e^4} + \frac{1}{4}$

$= 0.245$ ✓

Did not evaluate to 3 decimal places. Only 1 mark lost if done in both parts.

ii) $\int_1^5 \frac{dx}{\sqrt{x}} = \int_1^5 x^{-1/2}\,dx$

$= \left[2x^{1/2}\right]_1^5$ ✓ Calc - 4

$= \left[2\sqrt{x}\right]_1^5 = 2\sqrt{5} - 2\sqrt{1}$

$= 2.472$ ✓

Identify $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$. Some careless errors with simplifying

c) i) exterior angle of a triangle equals the sum of the two opposite interior angles ✓ Comm - 1

ii) $\angle DAC = 180 - (\alpha+\beta+\alpha)$ (angle sum of a triangle is 180°)

$= 180 - (2\alpha+\beta)$ ✓ Comm - 1

iii) $\angle ADE = \frac{180 - \angle DAC}{2}$

$= \frac{180 - [180 - (2\alpha+\beta)]}{2}$ ✓

$= \frac{2\alpha+\beta}{2}$

$= \alpha + \frac{\beta}{2}$

$\therefore \angle EDC = \angle ADC - \angle ADE$

$= \alpha+\beta - \left(\alpha+\frac{\beta}{2}\right)$

$= \beta - \frac{\beta}{2}$ Reas - 2

$= \frac{\beta}{2}$ ✓

i) Needed to say the wo[rd] opposite to get the ma[rk]. Other variations of the answer the mark was awarded as long as it was explained.
ii) Well done as long as correct reason was giv[en] and it was attempted.
iii) In terms of $\beta$ mea[ns] the solution should ha[ve] this term.
A lot of non-attempts. 'Hence' indicates one can use earlier parts to help with the soluti[on].

Q4 a) i) No of seats $= \underbrace{37+35+33+\ldots}_{\text{this is an A.P.}}$

with $a = 37$ ✓
$d = -2$

$\therefore T_n = a + (n-1)d$

$= 37 + (n-1)\times -2$

$= 37 - 2n + 2$

$= 39 - 2n$ ✓ Comm - 2

$\therefore$ $n^{th}$ row has $39-2n$ seats

Well done - be carefu[l] $d = -2$

ii) find $n$ $T_n \geq 0$

$39 - 2n \geq 0$

$2n \leq 39$ Reas - 1

$n \leq 19.5$

$\therefore$ greatest value $n$ can be is 19. ✓

Be careful with greatest value of $n$

iii) find $n$ when $S_n = 360$

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore 360 = \frac{n}{2}[2 \times 37 + (n-1) \times -2]$ ✓

$720 = n(74 - 2n + 2)$

$720 = n(76 - 2n)$

$720 = 76n - 2n^2$

$2n^2 - 76n + 720 = 0$

$n^2 - 38n + 360 = 0$

$(n-18)(n-20) = 0$ ✓

$n = 18 \quad n = 20$

since the greatest value for $n$ is 19

then $n = 18$

$\therefore$ 18 rows of seats. ✓ Reas - 3

- Learn formula

Be careful of substitution.

Algebra - factorising let alot of students down

b) i)

| $x$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $y = \ln x$ | 0.693 | 1.099 | 1.386 | 1.609 |

✓

Well done

ii) $\int_2^5 \ln x \, dx \doteqdot \frac{3-2}{2}[f(2) + f(3)] + \frac{4-3}{2}[f(3) + f(4)] + \frac{5-4}{2}[f(4) + f(5)]$ ✓

$= \frac{1}{2}[f(2) + 2 \times f(3) + 2f(4) + f(5)]$

$= \frac{1}{2}[0.693 + 2 \times 1.099 + 2 \times 1.386 + 1.609]$ ✓

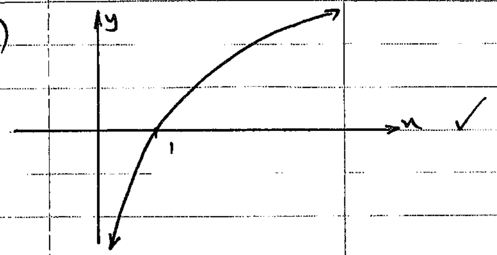
$= 3.6$ (to 1 d.p.) ✓ Calc - 3

Don't confuse Trapezoidal rule with Simpson's

2×rest does not mean $2 \times 1.386 \times 1.609$

One decimal place.

iii)




✓

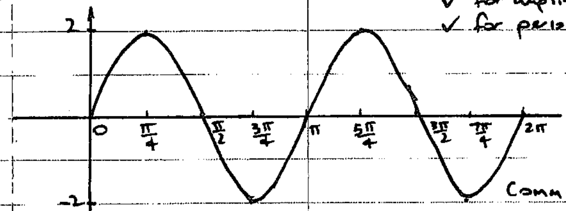
the graph of $y = \ln x$ is concave down, therefore the area of the trapezia will be less than the actual area. This leads to an underestimation of the integral. ✓ Comm - 2

Most students could sketch this

Students who made reference to their sketch or displayed their reason on their sketch got the mark. Must be trapezia though!

Q5 a) i)

✓ for amplitude
✓ for period




Comm - 2

You must clearly label the range values on the y-axis. Don't assume the lines on your page count as one unit.

ii) $A = \int_0^{\frac{\pi}{2}} 2\sin 2x \, dx + \left| \int_{\frac{\pi}{2}}^{\frac{3\pi}{4}} 2\sin 2x \, dx \right|$ ✓

$= \left[-\cos 2x\right]_0^{\frac{\pi}{2}} + \left| \left[-\cos 2x\right]_{\frac{\pi}{2}}^{\frac{3\pi}{4}} \right|$ ✓

$= (-\cos\pi + \cos 0) + \left| -\cos\frac{3\pi}{2} + \cos\pi \right|$

$= (1+1) + |0 - 1|$

$= 3 \text{ units}^2$ ✓ Calc - 3

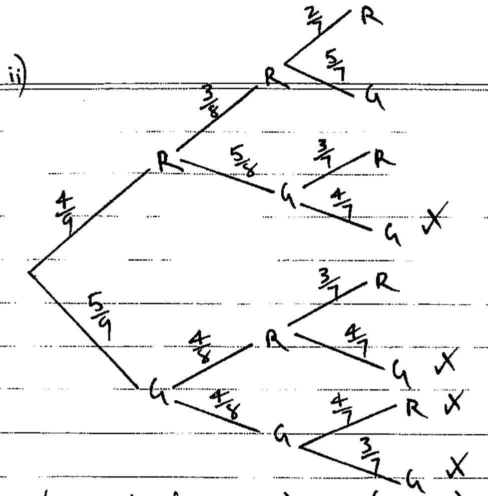
Poorly done. The total area should be split into pieces with absolute value around any area that lies below the x-axis.

Check the standard integrals when finding $\int 2\sin 2x \, dx$

b) i) $P(GRG) = \frac{5}{9} \times \frac{4}{8} \times \frac{4}{7}$

$= \frac{10}{63}$ ✓

ii)




$P(\text{Majority of green}) = P(2G\,1R) + P(3G)$

$= 3\times\left(\frac{5}{9}\times\frac{4}{8}\times\frac{4}{7}\right) + \frac{5}{9}\times\frac{4}{8}\times\frac{3}{7}$ ✓

$= 3\times\frac{10}{63} + \frac{5}{42}$

$= \frac{25}{42}$ ✓

Comm - 2

The students who drew a clear tree diagram had the greatest success in this part. Don't forget $0 \le P(E) \le 1$ so think about the validity of your answer.

Calculator errors not penalised here but if you got it wrong, please practise your calculator steps.

c) i) $f(e^2) = \log_e \sqrt{e^2}$

$= \log_e e$

$= 1$ ✓

ii) $y = \log_e \sqrt{x}$

$= \frac{1}{2}\log_e x$ ✓

$\frac{dy}{dx} = \frac{1}{2}\times\frac{1}{x}$

$= \frac{1}{2x}$ ✓

$\therefore m_{tang} = \frac{1}{2e^2}$ at $(e^2, 1)$

$\therefore$ using $y - y_1 = m(x - x_1)$

$y - 1 = \frac{1}{2e^2}(x - e^2)$

$2e^2 y - 2e^2 = x - e^2$

$x - 2e^2 y + e^2 = 0$ ✓

Calc - 3

Very poorly done, which was surprising.

$\frac{d}{dx}\left(\log_e f(x)\right) = \frac{1}{f(x)} \times f'(x)$

$\frac{d}{dx}\left(\log_e x^{1/2}\right) = \frac{1}{x^{1/2}} \times \frac{1}{2}x^{-1/2}$

$= \frac{1}{2x}$

Q6 a) i) $y = x^3 - 3x$

$\frac{dy}{dx} = 3x^2 - 3$

$\therefore$ for stationary points $\frac{dy}{dx} = 0$

$3x^2 - 3 = 0$

$3(x^2 - 1) = 0$

$3(x-1)(x+1) = 0$ ✓

$x = 1 \quad x = -1$

$y = -2 \quad y = 2$

$\frac{d^2y}{dx^2} = 6x$

test $(1,-2)$ $\frac{d^2y}{dx^2} = 6 > 0$

$\therefore (1,-2)$ is a min. t.p ✓

test $(-1,2)$ $\frac{d^2y}{dx^2} = -6 < 0$

$\therefore (-1,2)$ is a max. t.p. ✓

Calc - 3

Several students did n[ot] find the y-coordinate. A point requires a pair of coordinates.

ii) when $x = 0$ $\frac{d^2y}{dx^2} = 6\times 0 = 0$

| $x$ | $-1$ | $0$ | $1$ |
|---|---|---|---|
| $\frac{d^2y}{dx^2}$ | $-6$ | $0$ | $6$ |

Calc - II

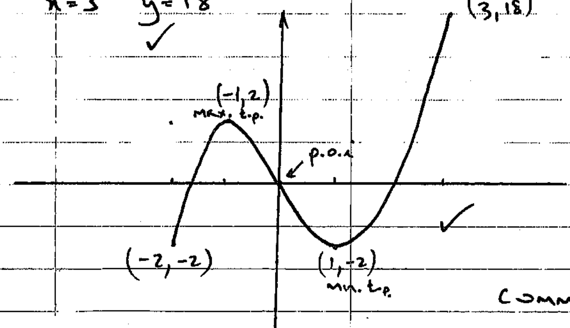
$\therefore$ change of concavity ✓

$\therefore (0,0)$ is a point of inflexion.

Many students appear to not understand that the second derivative shows concavity. Students who write $x$ $-1$ $0$ $1$ / $\frac{d^2y}{dx^2}$ $-6$ $0$ $6$ This is not a p.o.i. shows poor understanding and make the examiner very suspicious.

iii) $x = -2 \quad y = -2$

$x = 3 \quad y = 18$ ✓




Comm - 2

Students must resist the temptation to add extra p.o.i's in their graph. Always draw smooth curv[e] not a straight line. Alwa[ys] evaluate endpoints. Do diagrams in pencil.

iv) Since $y = x^3 - 3x$ is an odd function

$$\int_{-1}^{1} x^3 - 3x \, dx = 0 \checkmark$$

Reas - 1

Very few students understood the difference between an integral and an area. Most students incorrectly found the area.

b) Amount eaten $= \underbrace{5 + 3.75 + 2.8125 + \ldots}_{GP}$ $a = 5$, $r = \frac{3}{4}$ ✓

$$\therefore S_\infty = \frac{5}{1 - \frac{3}{4}}$$

$$= 20$$

Reas - 2

$\therefore$ Since $20 < 25$ Cat will not finish the leaf. ✓

Best answers used limiting sum other answers that showed supporting calculations were acceptable

c) i) $\frac{d}{dx}\left(e^{\tan 2x}\right)$

$$= 2\sec^2 2x \, e^{\tan 2x} \checkmark$$

Many students did not pay adequate attention to detail and omitted 2 in $2\sec^2 2x \, e^{\tan 2x}$

ii) $\int_0^{\frac{\pi}{6}} (\sec^2 2x) e^{\tan 2x} \, dx$

$$= \frac{1}{2}\int_0^{\frac{\pi}{6}} (2\sec^2 2x) e^{\tan 2x} \, dx$$

$$= \frac{1}{2} e^{\tan 2x} \Big]_0^{\pi/6} \checkmark$$

$$= \frac{1}{2}\left(e^{\tan \pi/3} - e^{\tan 0}\right)$$

$$= \frac{1}{2}\left(e^{\sqrt{3}} - e^0\right)$$

$$= \frac{e^{\sqrt{3}} - 1}{2} \checkmark$$

Calc - 3

Many could not relate the parts and incorrectly found $2e^{\tan 2x}$ instead of $\frac{1}{2}e^{\tan 2x}$.

Q7 a) i) $y^2 - 2y - 4x - 7 = 0$

$$y^2 - 2y = 4x + 7$$

$$y^2 - 2y + 1 = 4x + 8 \checkmark$$

$$(y-1)^2 = 4(x+2)$$

$\therefore$ vertex $(-2, 1)$ ✓

Careless mistakes with completing the square. Some students had no idea

ii) $4a = 4$

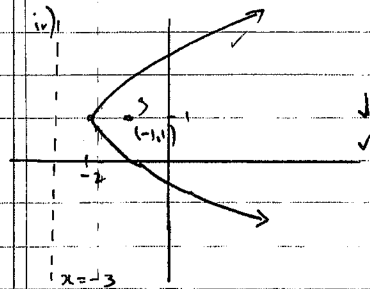
$a = 1$

$\therefore$ focus $(-1, 1)$ ✓

Marks carried forward from mistakes made in i)

iii) eqn. of directrix $x = -3$ ✓

iv)




✓ for 3 features
✓ for graph

Comm - 2

Must label 3 features and have correct direction of graph.

b) i) to find the point of intersection solve simultaneously $y = e^{-x}$
$y = e^x - 1$

$$\therefore e^{-x} = e^x - 1 \checkmark$$

$$\frac{1}{e^x} = e^x - 1$$

$$1 = e^{2x} - e^x$$

$$e^{2x} - e^x - 1 = 0 \checkmark$$

Comm - 2

First mark most students got, however most 'fudged' to show equation resulting in no 2nd mark. No idea what to do after letting 2 equations equal each other.

ii) let $m = e^x$

$$m^2 - m - 1 = 0$$

$$\therefore m = \frac{-(-1) \pm \sqrt{(-1)^2 - 4\times 1 \times -1}}{2\times 1} \quad ✓$$

$$= \frac{1 \pm \sqrt{1+4}}{2}$$

$$= \frac{1 \pm \sqrt{5}}{2}$$

since $e^x > 0$

then $e^x = \frac{1+\sqrt{5}}{2}$

$$\therefore x = \ln\left(\frac{1+\sqrt{5}}{2}\right) \quad ✓$$

Recs - 2

$$= 0.481$$

Basic use of '-b' formula + many students made careless errors

only one soln. as $e^x > 0$

students did not find $\ln\left(\frac{1+\sqrt{5}}{2}\right)$

iii) $$A = \int_0^{0.481} e^{-x} - e^x + 1 \, dx$$

$$= -e^{-x} - e^x + x \Big]_0^{0.481} \quad ✓$$

$$= \left(-e^{-0.481} - e^{0.481} + 0.481\right) - \left(-e^0 - e^0 + 0\right)$$

$$= 0.2 \text{ units}^2 \quad ✓$$

Calc - 2

This part could be completed without answers above parts correctly

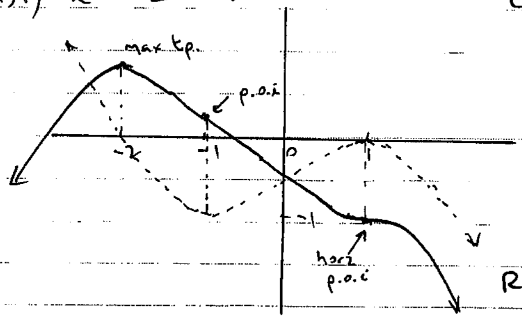
Be careful of signs

$e^{-x} - (e^x - 1)$

$= e^{-x} - e^x + 1$

Learn how to integrate exponentials.

Q8 a) i) $x < -2$ ✓ Calc - 1




Recs - 3

✓ for correct concavity

✓✓ for important features

Need to be careful with accuracy of the sketches.

A few students drew the point of inflexion at $x = -1$ as a horizontal p.o.i.

A few students drew the derivative graph.

b) i) for no real roots $\Delta < 0$

$$\Delta = b^2 - 4ac$$

$$= (k+3)^2 - 4\times k \times -1 \quad ✓$$

$$= k^2 + 6k + 9 + 4k$$

$$= k^2 + 10k + 9$$

$$\therefore k^2 + 10k + 9 < 0 \quad ✓$$

$$(k+9)(k+1) < 0$$

$$\therefore -9 < k < -1 \quad ✓$$

Comm - 3

Done fairly well. Although some students solved $(k+9)(k+1) <$ as $k < -1$ $k < -9$

ii) for positive definite $\Delta < 0$ and $a > 0$

now $a = k$

since $k$ has to be between $-9$ and $-1$

for $\Delta < 0$ it will never be positive

$\therefore a \not> 0$ ✓ Comm - 1

$\therefore$ it can never be positive definite

Very poorly explained. Most students didn't give enough detail. You had to relate a to k.

c) i) $x = \frac{\pi}{8}$ $y = \sec\frac{\pi}{4}$

$$= \frac{1}{\cos\frac{\pi}{4}}$$

$$= \frac{1}{\frac{1}{\sqrt{2}}}$$

$$= \sqrt{2} \quad ✓$$

$$\therefore A\left(\frac{\pi}{8}, \sqrt{2}\right)$$

Done well!

ii) Volume of solid = Volume of disc $- \pi\int_a^b [f(x)]^2 \, dx$

$$= \pi r^2 h - \pi\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$$

$$= \pi \times (\sqrt{2})^2 \times \frac{\pi}{8} - \pi\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx \quad ✓$$

$$= \frac{\pi^2}{4} - \pi\int_0^{\frac{\pi}{8}} \sec^2 2x \, dx$$

Most students didn't know how to derive this equation, but it was pleasing to see that they then went on to get the correct volume

$= \frac{\pi^2}{4} - \pi\left[\frac{1}{2}\tan 2x\right]_0^{\pi/8}$ ✓

$= \frac{\pi^2}{4} - \pi\left[\frac{1}{2}\tan\frac{\pi}{4} - \frac{1}{2}\tan 0\right]$

$= \frac{\pi^2}{4} - \pi\left(\frac{1}{2} - 0\right)$

$= \frac{\pi^2}{4} - \frac{\pi}{2}$

$= \frac{\pi^2 - 2\pi}{4}$ units$^3$ ✓ Calc - 3

Q9 a) $2\sin^2 x - 3\cos x = 0$

$2(1-\cos^2 x) - 3\cos x = 0$ ✓

$2 - 2\cos^2 x - 3\cos x = 0$

$2\cos^2 x + 3\cos x - 2 = 0$

$(2\cos x - 1)(\cos x + 2) = 0$

$\cos x = \frac{1}{2}$ $\quad \cos x = -2$ ✓

no solution as $-1 \le \cos x \le 1$

$x = \frac{\pi}{3}$ or $x = \frac{5\pi}{3}$ ✓ Reas - 3

Alot of variations for $\sin^2 x$

Be careful to answer $0 \le x \le 2\pi$.

b) i) $r^2 + h^2 = 9^2$ by Pythagoras' Thm

$r^2 = 81 - h^2$ ✓

$r = \sqrt{81-h^2}$ as $r \ge 0$

Very easy mark

When asked to show don't 'fudge' to get answer.

ii) Volume of solid $= \pi r^2 h + \frac{1}{3}\pi r^2 h$ ✓

$= \frac{4}{3}\pi r^2 h$

$= \frac{4}{3}\pi(81-h^2)h$ ✓ Comm - 2

$= 108\pi h - \frac{4}{3}\pi h^3$

Alot of students did not know the volume of cone formula.

iii) Max volume occurs when $\frac{dV}{dh} = 0$

$\frac{dV}{dh} = 108\pi - 4\pi h^2$ ✓

Most students carried forward ii) correctly to gain this mark!

$\therefore$ $108\pi - 4\pi h^2 = 0$

$4\pi h^2 = 108\pi$

$h^2 = 27$

$h = \pm 3\sqrt{3}$

since h represents a height

$h = 3\sqrt{3}$ ✓

test $\frac{d^2V}{dh^2} = -8\pi h$

when $h = 3\sqrt{3}$

$\frac{d^2V}{dh^2} = -8\pi \times 3\sqrt{3}$

$= -24\sqrt{3}\pi < 0$ ✓ Calc - 3

$\therefore h = 3\sqrt{3}$ gives a maximum volume.

Be careful solving for h - careless errors being made.

Test V'' to test the h for max. volume. Remember h is a positive value.

c) i) $P(3W) = \left(\frac{1}{50}\right)^3$ ✓

$= \frac{1}{125000}$

Well done

ii) P(at least one win) = 1 − P(no win)

$\therefore$ find n such that

$1 - \left(\frac{49}{50}\right)^n \ge 0.5$ ✓

$\left(\frac{49}{50}\right)^n \le 0.5$

$\ln\left(\frac{49}{50}\right)^n \le \ln(0.5)$

$n\ln\left(\frac{49}{50}\right) \le \ln(0.5)$

$n \ge \frac{\ln(0.5)}{\ln\left(\frac{49}{50}\right)}$

$\ge 34.3096\ldots$

$\therefore n = 35$

$\therefore$ it would require 35 draws. ✓

Reas - 2

Alot of students didn't know how to set this up.

Very similar question given to you in the last assessment.

If you are coming out with a negative solution "Alarm bells" should be sounding, some mistakes being made!

Q10 a) i) $\angle AOB = \frac{2\pi}{8}$

$= \frac{\pi}{4}$

$\therefore$ Area of $\triangle AOB = \frac{1}{2}\times1\times1\times\sin\frac{\pi}{4}$ ✓

$= \frac{1}{2\sqrt{2}}\ cm^2$

$\therefore$ area of the octagon $= 8\times\frac{1}{2\sqrt{2}}$

$= \frac{4}{\sqrt{2}}$

$= 2\sqrt{2}\ cm^2$ ✓

Most students didn't realise they needed $A=\frac{1}{2}ab\sin C$. Many students used $A=\frac{1}{2}r^2(\theta-\sin\theta)$

ii) $\angle MOP = \frac{\pi}{8}$ (as $\triangle POS$ is isosceles and OM is an altitude)

$\therefore \tan\frac{\pi}{8} = \frac{PM}{OM}$

$= \frac{PM}{1}$

$\therefore PM = \tan\frac{\pi}{8}$

$\therefore$ Area $\triangle POS = \frac{1}{2}\times b\times h$

$= \frac{1}{2}\times 2\tan\frac{\pi}{8}\times 1$ ✓

$= \tan\frac{\pi}{8}$

$\therefore$ Area of the octagon is $8\tan\frac{\pi}{8}$ ✓

Most students didn't realise right angle trig was required.

iii) Area of small octagon < area of circle < area of large octagon

$2\sqrt{2} < \pi\times1^2 < 8\tan\frac{\pi}{8}$ ✓

$2\sqrt{2} < \pi < 8\tan\frac{\pi}{8}$ Recs - 5

$\sqrt{2} < \frac{\pi}{2} < 4\tan\frac{\pi}{8}$.

b) 9.12% p.a = 0.76% pm

$\therefore A_1 = 400\,000 + \frac{0.76}{100}\times 400\,000 - M$

$= 400\,000(1.0076) - M$

$A_2 = A_1 + \frac{0.76}{100}A_1 - M$

$= A_1(1.0076) - M$

$= [400\,000(1.0076) - M](1.0076) - M$

$= 400\,000(1.0076)^2 - 1.0076M - M$ ✓

When the question says derive you need to show the relationship between $A_1$, $A_2$ and $A_3$ ie. establish a pattern.

$= 400\,000(1.0076)^2 - M(1+1.0076)$

$A_3 = A_2 + \frac{0.76}{100}A_2 - M$

$= A_2(1.0076) - M$

$= [400\,000(1.0076)^2 - 1.0076M - M]1.0076 - M$

$= 400\,000(1.0076)^3 - 1.0076^2M - 1.0076M - M$

$= 400\,000(1.0076)^3 - M(1+1.0076+1.0076^2)$

continuing the pattern

$\therefore A_{300} = 400\,000(1.0076)^{300} - M(1+1.0076+\dots+1.0076^{299})$ ✓

Comm - 2

ii) $A_{300} = 0$

$\therefore 400000(1.0076)^{300} - M(\underbrace{1+1.0076+\dots+1.0076^{299}}_{GP:\ a=1,\ r=1.0076,\ n=300}) = 0$

$400\,000(1.0076)^{300} - M\times 1\left(\frac{1.0076^{300}-1}{1.0076-1}\right) = 0$ ✓

$\therefore M\left(\frac{1.0076^{300}-1}{0.0076}\right) = 400\,000(1.0076)^{300}$

$M = 400\,000(1.0076)^{300}\times\frac{0.0076}{(1.0076^{300}-1)}$

$= 3389.72$

$\therefore$ monthly repayments are \$3389.72 ✓

Recs - 2

Done very well! Many students know how to manipulate the $S_n$ formula to get the correct value of M.

iii) find $A_{120}$ when $M = 3389.72$

$\therefore A_{120} = 400\,000(1.0076)^{120} - 3389.72(1+1.0076+\dots+1.0076^{119})$

$= 400\,000(1.0076)^{120} - 3389.72\times 1.\left(\frac{1.0076^{120}-1}{1.0076-1}\right)$

$= \$331\,863$ (to nearest dollar) ✓

Comm - 1

Reasonably well done for those who reached it!

iv) $12\%$ p.a. $= 1\%$ p.m.

$$A_n = 331863(1.01)^n - 3800(1+1.01+\dots+1.01^{n-1})$$
$$= 331863(1.01)^n - 3800 \times 1\frac{(1.01^n-1)}{1.01-1} \checkmark$$
$$= 331863(1.01)^n - 380000(1.01^n-1)$$

now $A_n = 0$

$$\therefore \quad 331863(1.01)^n - 380000(1.01^n-1) = 0$$
$$331863(1.01)^n - 380000(1.01)^n + 380000 = 0$$
$$48137(1.01)^n = 380000$$
$$(1.01)^n = 7.894\dots$$
$$n = \frac{\ln(7.894\dots)}{\ln(1.01)}$$
$$= 207.64\dots \checkmark$$

It would require a further 208 months

Recs - 2

For the students who attempted this question, silly numerical errors made it difficult to arrive at the correct value.